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Connection between proton decay suppression and seesaw mechanism in supersymmetric SO(10) models

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ABSTRACT: We propose a mechanism to suppress proton decay induced by dimension-5 operators in a supersymmetric SO(10) model. Proton lifetime is directly connected with the intermediate vacuum expectation value which is responsible for the seesaw mechanism. The model shows many consistencies with the present theoretical results such as the components of the two Higgs doublets in the minimal supersymmetric standard model.

KEYWORDS: GUT, proton decay, seesaw mechanism, fermion masses and mixing

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1 Introduction

The Grand Unified Theory (GUT) [1, 2] has never failed to fascinate the particle physicists since it was proposed. Besides unifying the gauge interactions in a simple group, it also unifies quarks and leptons in same multiplets which, however, are very different in $SU(3)_C \times SU(2)_L \times U(1)_Y$ of the standard model (SM). Consequently, lepton and baryon numbers are not conserved separately. While lepton number violation is strongly supported by the observation of neutrino oscillations which are usually explained through the seesaw mechanism [3–11], the baryon number violation is strongly constrained by the proton decay experiments which need a natural explanation.

Supersymmetric (SUSY) GUT model based on $SO(10)$ [12, 13], among the various GUT models, is very attractive due to its several advantages. Firstly, as protected by supersymmetry, it has no problem in naturalness. Secondly, having all the fermions of a generation contained in one **16** dimensional representation which contains the right-handed neutrino, the model can naturally explain the neutrino oscillations through the seesaw mechanism. Thirdly, in the minimal [14–16] and the next-to-minimal [17, 18] versions of SUSY $SO(10)$, the theories are renormalizable and R -parity is conserved which prohibits the most dangerous dimension-4 operators for proton decay.

Nevertheless, the SUSY $SO(10)$ models have also difficulties to overcome. To realize the seesaw mechanism, an intermediate scale is usually introduced. This will generally bring in new particles at this scale and break down gauge coupling unification badly [16]. However, as noticed recently in [19], there is actually no need to introduce an intermediate seesaw scale above which new gauge interactions begin. Instead, the seesaw mechanism requires only an intermediate vacuum expectation value (VEV) which contributes only a small portion of the new gauge bosons whose masses are still around the GUT scale. As

a consequence, the gauge coupling unification will not be broken down as in those models, e.g., the minimal SUSY SO(10) model (MSSO10).

Furthermore, in the SUSY models, the dimension-5 operators dominate the proton decay rates and therefore strongly need to be suppressed by a mechanism. In the literature, since these operators are related to the Yukawa couplings, careful adjustments of the Yukawa couplings [20] are common which however are not sufficient as the lower limit on the proton lifetime from experiments is increasing.

In this work, instead of strictly solving the doublet-triplet splitting problem labored by many groups [21, 22], we simply assume that the Higgs doublet pair in the minimal supersymmetric standard model (MSSM) are achieved by fine-tuning which will not be performed explicitly. Our efforts are mainly focused on proposing a mechanism to sufficiently suppress the proton decay rates. We will extend the MSSO10 to achieve this goal. The effective triplet mass (ETM) [23, 24], to which the dimension-5 operators are inversely proportional, is enhanced due to the special structure of the color-triplet Higgs mass matrix. This suppression of proton decay is found to be directly related to the intermediate VEV required by the seesaw mechanism. We also find that the massless MSSM doublets obtained by the assumed fine-tuning are also related to the intermediate VEV, and that these doublets conform to the results from simply fitting the fermion sector in SO(10) models without considering other stringent constraints.

In the next section we will present the model, followed by the realization of the seesaw mechanism in Section 3. The solution of the model required by SUSY is presented in Section 4. The mechanism of suppression proton decay follows in Section 5. Predictions on the MSSM Higgs doublets are given in Section 6. We will summarize finally.

2 The present model

The present model contains the following particles in the spectrum. First, each generation of the matter superfields are contained in a **16**-plet superfields ψ_i ($i = 1, 2, 3$) as in most of the SO(10) models. Second, we use **210**-plet Higgs to break GUT symmetry. To further break $U(1)_R \times U(1)_{B-L}$ symmetry down to $U(1)_Y$, two pairs of **126**+ $\overline{\mathbf{126}}$ -plet Higgs (denoted by $\Delta_i + \overline{\Delta}_i, i = 1, 2$) are introduced. Two Higgs doublets in **10** ($H_{1,2}$), together with those in the **126**+ $\overline{\mathbf{126}}$ s, are used to break down the electroweak symmetry. Third, we will introduce a U(1) symmetry to differentiate these Higgs into those couple with the matter fields and those do not. These U(1) quantum numbers Q are listed in Table 1.

Table 1. SO(10) multiplets and their U(1) charges.

Charges	ψ_i	H_1	$\Delta_1/\overline{\Delta}_1$	H_2	$\Delta_2/\overline{\Delta}_2$	Φ	S
Q	$-1/2$	1	1	-1	-1	0	2

Here we will simply treat the U(1) symmetry as a global one broken by the VEV of a SO(10) singlet S which is taken as

$$S_0 \sim M_I \sim 10^{14} \text{GeV} \sim 10^{-2} M_G. \quad (2.1)$$

In Section 3 this VEV S_0 will naturally generate the seesaw VEV and thus the model has no mass larger than the GUT scale explicitly. The value S_0 in (2.1) is also of the order of $\frac{M_G^2}{M_{Planck}}$, which may suggest alternatively that it is possible to be realized through an analogue of a seesaw mechanism, if we treat the $U(1)$ as an anomalous symmetry broken by a Planck scale VEV generated by the Green-Schwarz mechanism [25–28]. For simplicity, this later possibility will not be discussed further.

The matter fields are negative in $U(1)$ charges, so the Yukawa superpotential is

$$W_Y = Y_{10}^{ij} \psi_i \psi_j H_1 + Y_{126}^{ij} \psi_i \psi_j \bar{\Delta}_1, \quad (2.2)$$

which is just the same as in the MSSO10. The most general renormalizable superpotential in the Higgs sector is given by

$$\begin{aligned} W_H = & \frac{1}{2} m_\Phi \Phi^2 + m_{\Delta 12} \bar{\Delta}_1 \Delta_2 + m_{\Delta 21} \bar{\Delta}_2 \Delta_1 + m_H H_1 H_2 \\ & + (\beta_{12} \Delta_1 + \bar{\beta}_{12} \bar{\Delta}_1) H_2 \Phi + (\beta_{21} \Delta_2 + \bar{\beta}_{21} \bar{\Delta}_2) H_1 \Phi \\ & + \lambda \Phi^3 + (\lambda_{12} \bar{\Delta}_1 \Delta_2 + \lambda_{21} \bar{\Delta}_2 \Delta_1) \Phi + S \left(\frac{1}{2} \alpha_1 H_2^2 + \alpha_2 \bar{\Delta}_2 \Delta_2 \right). \end{aligned} \quad (2.3)$$

3 On the seesaw mechanism

The small but non-vanishing neutrino masses can be naturally explained using the seesaw mechanism. In a model where the type-I seesaw dominates, the mass matrix of neutrinos is given as $M_\nu \simeq -M_{\nu D}^T M_{\nu R}^{-1} M_{\nu D}$. The Majorana mass matrix $M_{\nu R}$ comes from the VEV of a $SU(2)_R$ triplet contained in $\mathbf{126}$, which corresponds to the seesaw scale M_I . A sub-eV neutrino mass roughly indicates $M_I \sim 10^{14} \text{ GeV} \sim 10^{-2} M_G$. However, the presence of an intermediate scale breaks the unification of gauge couplings badly [16] in general.

In the present model, the presence of two $\mathbf{126}$ s changes the situation and the GUT symmetry would be broken down to the SM symmetry directly. Instead of an intermediate scale, only an intermediate valued VEV, i.e. \bar{v}_{1R} , of the order $O(M_I)$ is required to couple with the matter fields [19]. The D-flatness required by SUSY at high energy scales is

$$|v_{1R}|^2 + |v_{2R}|^2 = |\bar{v}_{1R}|^2 + |\bar{v}_{2R}|^2, \quad (3.1)$$

where the v s and \bar{v} s are the VEVs of the $SU(2)_R$ triplets in $\mathbf{126}$ s and $\mathbf{126}$ s, respectively. Eq. (3.1) can be fulfilled even if \bar{v}_{1R} is small compared to the GUT scale. Then the seesaw mechanism does not conflict with gauge coupling unification if the other VEVs are taken at the GUT scale.

4 SUSY preserving at high energy

When the $SO(10)$ breaks down to the MSSM, only the MSSM singlets can get VEVs,

$$\begin{aligned} \Phi_1 &= \langle \Phi(1, 1, 1) \rangle, \quad \Phi_2 = \langle \Phi(15, 1, 1) \rangle, \quad \Phi_3 = \langle \Phi(15, 1, 3) \rangle; \\ v_{(1,2)R} &= \langle \Delta_{(1,2)}(\mathbf{10}, 1, 3) \rangle, \quad \bar{v}_{(1,2)R} = \langle \bar{\Delta}_{(1,2)}(10, 1, 3) \rangle. \end{aligned} \quad (4.1)$$

The Pati-Salam ($SU(4)_C \times SU(2)_L \times SU(2)_R$) subgroup indices are used to specify different singlets of the MSSM. Substituting these VEVs into (2.3), we get

$$\begin{aligned} \langle W_H \rangle = & \frac{1}{2} m_\Phi (\Phi_1^2 + \Phi_2^2 + \Phi_3^2) + \lambda \left(\frac{1}{9\sqrt{2}} \Phi_2^3 + \frac{1}{2\sqrt{6}} \Phi_1 \Phi_3^2 + \frac{1}{3\sqrt{2}} \Phi_2 \Phi_3^2 \right) + m_{\Delta 12} \bar{v}_{1R} v_{2R} \\ & + m_{\Delta 21} \bar{v}_{2R} v_{1R} + (\lambda_{12} \bar{v}_{1R} v_{2R} + \lambda_{21} \bar{v}_{2R} v_{1R}) \Phi_0 + \alpha_2 S_0 \bar{v}_{2R} v_{2R}, \end{aligned} \quad (4.2)$$

where we have defined

$$\Phi_0 = \left[\Phi_1 \frac{1}{10\sqrt{6}} + \Phi_2 \frac{1}{10\sqrt{2}} + \Phi_3 \frac{1}{10} \right].$$

In the presence of all the VEVs in (4.1), to preserve SUSY at high energy, besides the D-flatness condition in (3.1), the F-flatness conditions are also required, i.e.,

$$\left\{ \frac{\partial}{\partial \Phi_1}, \frac{\partial}{\partial \Phi_2}, \frac{\partial}{\partial \Phi_3}, \frac{\partial}{\partial v_{1R}}, \frac{\partial}{\partial \bar{v}_{1R}}, \frac{\partial}{\partial v_{2R}}, \frac{\partial}{\partial \bar{v}_{2R}} \right\} \langle W_H \rangle = 0. \quad (4.3)$$

Then we get

$$\begin{aligned} 0 = & m_\Phi \Phi_1 + \frac{\lambda \Phi_3^2}{2\sqrt{6}} + \frac{1}{10\sqrt{6}} (\lambda_{12} \bar{v}_{1R} v_{2R} + \lambda_{21} \bar{v}_{2R} v_{1R}), \\ 0 = & m_\Phi \Phi_2 + \frac{\lambda \Phi_2^2}{3\sqrt{2}} + \frac{\lambda \Phi_3^2}{3\sqrt{2}} + \frac{1}{10\sqrt{2}} (\lambda_{12} \bar{v}_{1R} v_{2R} + \lambda_{21} \bar{v}_{2R} v_{1R}), \\ 0 = & m_\Phi \Phi_3 + \frac{\lambda \Phi_1 \Phi_3}{\sqrt{6}} + \frac{\sqrt{2} \lambda \Phi_2 \Phi_3}{3} + \frac{1}{10} (\lambda_{12} \bar{v}_{1R} v_{2R} + \lambda_{21} \bar{v}_{2R} v_{1R}), \end{aligned} \quad (4.4)$$

for $\Phi_{1,2,3}$, respectively. The condition for v_{1R} and v_{2R} is

$$\begin{pmatrix} \bar{v}_{1R} & \bar{v}_{2R} \end{pmatrix} \begin{pmatrix} 0 & M_{12} \\ M_{21} & \alpha_2 S_0 \end{pmatrix} = 0, \quad (4.5)$$

and that for \bar{v}_{1R} and \bar{v}_{2R} is

$$\begin{pmatrix} 0 & M_{12} \\ M_{21} & \alpha_2 S_0 \end{pmatrix} \begin{pmatrix} v_{1R} \\ v_{2R} \end{pmatrix} = 0. \quad (4.6)$$

Here for simplicity we defined

$$M_{12} = m_{\Delta 12} + \lambda_{12} \Phi_0 \quad M_{21} = m_{\Delta 21} + \lambda_{21} \Phi_0. \quad (4.7)$$

Equations (4.5) and (4.6) both require

$$\text{Det} \begin{pmatrix} 0 & M_{12} \\ M_{21} & \alpha_2 S_0 \end{pmatrix} = M_{12} M_{21} = 0. \quad (4.8)$$

If we check the mass matrix of the SM singlets, we can see that under (4.8) the massless Goldstone mode responsible for $U(1)_{I_{3R}} \times U(1)_{B-L} \rightarrow U(1)_Y$ can be generated while all the other eigenstates in the same SM representation remain massive.

If we take $M_{21} = 0$, we can get the following solutions

$$-\frac{\bar{v}_{1R}}{\bar{v}_{2R}} = \frac{\alpha_2 S_0}{M_{12}} \sim 10^{-2}, \quad v_{2R} = 0, \quad (4.9)$$

which means we can naturally get the seesaw VEV \bar{v}_{1R} by considering the F-flatness conditions because of the intermediate VEV S_0 . It is not that both \bar{v}_{1R} and \bar{v}_{2R} get independent VEV, but only a combination of them gets VEV whose main component comes from \bar{v}_{2R} . For a vanishing v_{2R} , we define $\Phi_3 = 6m_\Phi x/\lambda$ following [16] and get

$$\begin{aligned} \Phi_1 &= -\frac{\sqrt{6}m_\Phi}{\lambda} \frac{x(1-5x^2)}{(1-x)^2}, \\ \Phi_2 &= -\frac{3\sqrt{2}m_\Phi}{\lambda} \frac{(1-2x-x^2)}{(1-x)}, \\ \lambda_{21}\bar{v}_{2R}v_{1R} &= \frac{60m_\Phi^2}{\lambda} \frac{x(1-3x)(1+x^2)}{(1-x)^2}. \end{aligned} \quad (4.10)$$

The x is then determined by $M_{21} = 0$ and thus determines $\bar{v}_{2R} \sim v_{1R}$ which are generally at the GUT scale.

If a vanishing M_{12} after (4.8) is taken instead, we can not get the wanted seesaw VEV and the further results of fermion masses are inconsistent with experiments. For these reasons, the $M_{12} = 0$ case will not be discussed further below.

In summary, SUSY at high energy and the seesaw mechanism choose to satisfy

$$\bar{v}_{1R} = M_I, \quad v_{2R} = 0, \quad (4.11)$$

for the $\text{SO}(10)$ symmetry breaking and thus

$$|\bar{v}_{2R}| \sim |v_{1R}| = \sqrt{\left| \frac{60m_\Phi^2}{\lambda\lambda_{21}} \frac{x(1-3x)(1+x^2)}{(1-x)^2} \right|} \quad (4.12)$$

following (4.10). All Higgs superfields are given masses at the GUT scale except the two doublets in MSSM whose masses require a minimal fine-tuning of the parameters as done in the MSSO10 [16]. Then gauge coupling unification can be realized by adjusting other parameters of the model.

5 The triplet mass matrix and suppression of proton decay

All the Higgs multiplets in Table 1 contain color triplet-antitriplet pairs. The color triplets are ordered as

$$\varphi_T = (H_{1T}, \Delta_{1T}, \bar{\Delta}_{1T}, \bar{\Delta}'_{1T}, \Phi_T, H_{2T}, \Delta_{2T}, \bar{\Delta}_{2T}, \bar{\Delta}'_{2T}), \quad (5.1)$$

while the color antitriplets are

$$\varphi_{\bar{T}} = (H_{1\bar{T}}, \bar{\Delta}_{1\bar{T}}, \Delta_{1\bar{T}}, \Delta'_{1\bar{T}}, \Phi_{\bar{T}}, H_{2\bar{T}}, \bar{\Delta}_{2\bar{T}}, \Delta_{2\bar{T}}, \Delta'_{2\bar{T}}). \quad (5.2)$$

The mass term of the Higgs color triplets is given by $(\varphi_{\overline{T}})_a(M_T)_{ab}(\varphi_T)_b$, with the 9×9 matrix M_T written as

$$M_T = \begin{pmatrix} B_{11(4 \times 4)} & B_{12(4 \times 5)} \\ B_{21(5 \times 4)} & B_{22(5 \times 5)} \end{pmatrix}. \quad (5.3)$$

The B_{11} is a 4×4 null matrix, and the rests are [17, 18]

$$B_{12} = \begin{pmatrix} \frac{\bar{\beta}_{21}\bar{v}_{2R}}{\sqrt{5}} & m_H & \beta_{21}\Phi_{H\Delta} & \bar{\beta}_{21}\Phi_{H\overline{\Delta}} & \frac{-\sqrt{2}\bar{\beta}_{21}\Phi_3}{\sqrt{15}} \\ 0 & \bar{\beta}_{12}\Phi_{H\Delta} & m_{\Delta_{12}} & 0 & 0 \\ \frac{-\lambda_{21}\bar{v}_{2R}}{10\sqrt{3}} & \beta_{12}\Phi_{H\overline{\Delta}} & 0 & m_{\Delta_{21}} & \frac{\lambda_{21}\Phi_3}{15\sqrt{2}} \\ \frac{-\lambda_{21}\bar{v}_{2R}}{5\sqrt{6}} & \frac{-\sqrt{2}\beta_{12}\Phi_3}{\sqrt{15}} & 0 & \frac{\lambda_{21}\Phi_3}{15\sqrt{2}} & M_{\Delta} \end{pmatrix}, \quad (5.4)$$

$$B_{21} = \begin{pmatrix} \frac{\beta_{21}v_{2R}}{\sqrt{5}} & 0 & \frac{-\lambda_{12}v_{2R}}{10\sqrt{3}} & \frac{-\lambda_{12}v_{2R}}{5\sqrt{6}} \\ m_H & \beta_{12}\Phi_{H\Delta} & \bar{\beta}_{12}\Phi_{H\overline{\Delta}} & -\bar{\beta}_{12}\frac{\sqrt{2}\Phi_3}{\sqrt{15}} \\ \bar{\beta}_{21}\Phi_{H\Delta} & m_{\Delta_{21}} & 0 & 0 \\ \beta_{21}\Phi_{H\overline{\Delta}} & 0 & m_{\Delta_{12}} & \frac{\lambda_{12}\Phi_3}{15\sqrt{2}} \\ -\beta_{21}\frac{\sqrt{2}\Phi_3}{\sqrt{15}} & 0 & \frac{\lambda_{12}\Phi_3}{15\sqrt{2}} & m_{\Delta_{12}} + \lambda_{12}\Phi_{\Delta} \end{pmatrix}, \quad (5.5)$$

and

$$B_{22} = \begin{pmatrix} M_{\Phi} & \frac{\beta_{12}v_{1R}}{\sqrt{5}} & 0 & \frac{-\lambda_{21}v_{1R}}{10\sqrt{3}} & \frac{-\lambda_{21}v_{1R}}{5\sqrt{6}} \\ \frac{\bar{\beta}_{12}\bar{v}_{1R}}{\sqrt{5}} & \alpha_1 S & 0 & 0 & 0 \\ 0 & 0 & \alpha_2 S & 0 & 0 \\ \frac{-\lambda_{12}\bar{v}_{1R}}{10\sqrt{3}} & 0 & 0 & \alpha_2 S & 0 \\ \frac{-\lambda_{12}\bar{v}_{1R}}{5\sqrt{6}} & 0 & 0 & 0 & \alpha_2 S \end{pmatrix}, \quad (5.6)$$

where for simplicity we have defined

$$\begin{aligned} \Phi_{H\Delta} &= -\frac{\Phi_1}{\sqrt{10}} + \frac{\Phi_2}{\sqrt{30}}, & M_{\Delta} &= m_{\Delta_{21}} + \lambda_{21}\Phi_{\Delta}, \\ \Phi_{H\overline{\Delta}} &= -\frac{\Phi_1}{\sqrt{10}} - \frac{\Phi_2}{\sqrt{30}}, & M_{\Phi} &= m_{\Phi} + \lambda\left(\frac{\Phi_1}{\sqrt{6}} + \frac{\Phi_2}{3\sqrt{2}} + \frac{2\Phi_3}{3}\right), \\ \Phi_{\Delta} &= \frac{\Phi_1}{10\sqrt{6}} + \frac{\Phi_2}{30\sqrt{2}}. \end{aligned} \quad (5.7)$$

The determinant of M_T is nonzero and consequently M_T is reversible with all eigenvalues at GUT scale.

In SUSY GUTs, the dominant mechanism inducing proton decays is through the dimension-5 operators [23, 24]

$$-W_5 = C_L^{ijkl} \frac{1}{2} q_i q_j q_k l_l + C_R^{ijkl} u_i^c d_j^c u_l^c e_k^c, \quad (5.8)$$

which are called the $LLLL$ and $RRRR$ operators, respectively, obtained by integrating out the colored triplet Higgs superfields in the interactions in (2.2). The coefficients C_L s at the GUT scale M_G are [29]

$$C_L^{ijkl}(M_G) = Y_{10}^{ij}(M_T^{-1})_{11} Y_{10}^{kl} + Y_{10}^{ij}(M_T^{-1})_{12} Y_{126}^{kl}$$

$$+ Y_{126}^{ij}(M_T^{-1})_{31}Y_{10}^{kl} + Y_{126}^{ij}(M_T^{-1})_{32}Y_{126}^{kl}. \quad (5.9)$$

The Yukawa couplings have been rather constrained by fitting the fermion masses and mixing, thus suppressing proton decay rates needs some detailed investigations on the matrix elements in M_T .

From (2.2), only H_1 and $\bar{\Delta}_1$ couple with fermions, and hence it is the up-left 4×4 block of M_T^{-1} that can affect the proton decay through the dimension-5 operators. These relevant elements in the M_T^{-1} are proportional to their corresponding algebraic complements divided by the determinant of M_T . These corresponding algebraic complements are proportional to $\bar{\nu}_{1R} \sim M_I$ or $S_0 \sim M_I$ in (5.6) which is small compared with the GUT scale. As a consequence, the proton decay amplitudes are suppressed by a factor M_I/M_G .

Now we have established in the present model a proportional relation between the intermediate VEV M_I , required by the seesaw mechanism, and the proton decay amplitudes. Consequently, the proton decay amplitudes are proportional to $\frac{M_I}{M_G^2}$, substantially suppressed compared to $\frac{1}{M_G}$ in the usual models. For the $RRRR$ type operators the results are just the same.

Relating the proton decay suppression with the seesaw VEV can be understood in other viewpoints. Since only part of the elements in the the up-left 4×4 block couple to the matter fields, we can get a smaller effective mass matrix by integrating out the down-right 5×5 block formally

$$M_{\text{eff}} = -B_{12} \cdot B_{22}^{-1} \cdot B_{21}. \quad (5.10)$$

From (5.6), B_{22} has only one GUT scale mass eigenvalue. Rotating the bases and transforming B_{22} into diagonal form,

$$D_{22} = \text{diag } O(M_G, M_I, M_I, M_I, M_I), \quad (5.11)$$

The elements of B_{11} remain to be zero, while those of B_{12} and B_{21} are still of the order $O(M_G)$, i.e.,

$$M_T \rightarrow \tilde{M}_T \simeq \begin{pmatrix} 0_{(4 \times 4)} & M_{G(4 \times 5)} \\ M_{G(5 \times 4)} & D_{22(5 \times 5)} \end{pmatrix}. \quad (5.12)$$

Indeed, each one of the four M_I eigenvalues in B_{22} gives rise to an eigenvalue of the order $O(\frac{M_G^2}{M_I})$ in M_{eff} . The largest one, M_G , contributes as corrections of the order $O(M_G)$ to the above four eigenvalues in M_{eff} and hence are negligible. In summary, it is the lightest eigenvalue in M_{eff} that dominates in proton decay, and it turns out to be

$$M_{HC}^{\text{eff}} \sim \frac{M_G^2}{M_I} \sim 2 \times 10^{18} \text{ GeV}. \quad (5.13)$$

For general values of parameters of $\text{SO}(10)$ GUTs, it is definitely sufficient to suppress the proton decay rates to satisfy the current experimental limits.

This mechanism of suppression of proton decay can be equivalently achieved by another method. The **210**-plet does not couple to the matter fields thus its color triplet-antitriplet components can be integrated out first. In result, the reduced mass matrix for the color triplet-antitriplet Higgs is now 8×8 whose four blocks are all 4×4 : (i) B_{11} keeps unchanged

as a null matrix; (ii) B_{12} has its leftmost column eliminated, while the other elements remain at M_G ; (iii) B_{21} has its uppermost row eliminated, while the other elements remain at M_G ; (iv) B_{22} has its leftmost column and lowest row eliminated, while the other elements are the order $O(\frac{v_{1R}}{M_\Phi} \bar{v}_{1R}) \sim M_I$. In the limit $M_I \rightarrow 0$, this structure is an analogue to the mass matrix for the Higgs color triplets in the flipped SU(5) model [30–33] which, as is well known, has negligible contributions of dimension-5 operators to proton decay. With the M_I elements kept, the inducing suppressed proton decay amplitudes would be of the order $O(\frac{M_I}{M_G^2})$, same as (5.13).

6 The doublets

To get the almost massless MSSM doublets H_u and H_d , we need a minimal fine-tuning in the mass matrix of the doublets. In the present model, we have Higgs doublets as follows:

$$\varphi_u = (H_{1u}, \Delta_{1u}, \bar{\Delta}_{1u}, \Phi_u, H_{2u}, \Delta_{2u}, \bar{\Delta}_{2u}), \quad (6.1)$$

$$\varphi_d = (H_{1d}, \bar{\Delta}_{1d}, \Delta_{1d}, \Phi_d, H_{2d}, \bar{\Delta}_{2d}, \Delta_{2d}). \quad (6.2)$$

After symmetry breaking at the GUT scale, only one pair of Higgs doublets remain massless, i.e.,

$$H_u = \sum_{i=1}^7 \alpha_u^{i*} \varphi_u^i, \quad H_d = \sum_{i=1}^7 \alpha_d^{i*} \varphi_d^i. \quad (6.3)$$

The mass matrix for the doublets is symbolically written as

$$M_D = \left(\begin{array}{c|c} 0_{(4 \times 3)} & M_{G(4 \times 4)} \\ \hline M_{G(3 \times 3)} & M_{I(3 \times 4)} \end{array} \right), \quad (6.4)$$

whose determinant factorizes into the determinant of $M_{G(3 \times 3)}$ times that of $M_{G(4 \times 4)}$. The existence of zero eigenvalue in M_D thus requires the determinant of either $M_{G(3 \times 3)}$ or $M_{G(4 \times 4)}$ is zero. In solving the eigenstates of M_D the order M_I entries can be taken as small perturbations. The solutions corresponding to $\text{Det}(M_{G(4 \times 4)}) = 0$ lead to small up-type quark masses which is excluded by the heavy top quark mass. The other solutions corresponding to $\text{Det}(M_{G(3 \times 3)}) = 0$ give, up to normalization factors,

$$\alpha_u^* = O(1, 1, 1, 0, 0, 0, 0), \quad (6.5)$$

$$\alpha_d^* = O(\frac{M_I}{M_G}, \frac{M_I}{M_G}, \frac{M_I}{M_G}, \frac{M_I}{M_G}, 1, 1, 1). \quad (6.6)$$

This just explains the large ratio of $\frac{m_t}{m_b}$, and further gives

$$\tan\beta = \frac{v_u}{v_d} \approx \frac{m_t}{m_b} \frac{M_I}{M_G} \sim O(1), \quad (6.7)$$

suggesting that a small $\tan\beta$ is favored in the present model. This indeed agrees with a similar result

$$\frac{\alpha_u^1}{\alpha_d^1} \tan\beta \sim 10^2, \quad \frac{\alpha_u^3}{\alpha_d^2} \tan\beta \sim 10^2, \quad (6.8)$$

got by simply fitting the fermion parameters in SO(10) models from many groups [15, 34–36] without considering other constraints. In the present model, the ratios on the R.H.S. of (6.8), however, are predicted to be related to the ratio $\frac{M_G}{M_I}$. Also, (6.5) holds exactly, showing that there is no Φ_u component in H_u which, following [37], suggests that it is the type-I instead of type-II seesaw mechanism that works in the present model.

7 Comments and conclusion

In this work we have proposed a SUSY SO(10) model for sufficient suppression of proton decay. The suppression is found to be linked with the intermediate VEV required by the seesaw mechanism. The seesaw mechanism turns out to be type-I. Assuming that the two doublets in MSSM are achieved by fine-tuning, we find the components of these doublets agree in magnitudes with those got by just fitting the fermion masses and mixing. Again, the ratios of components are linked to the ratio of the GUT scale versus the intermediate VEV. Since all the Higgs particles beyond the MSSM doublets are at GUT scale, the unification of coupling constants will be maintained by adjusting the parameters. Above the GUT scale, the gauge coupling of SO(10) will increase fast into the non-perturbative region, as many of the SUSY SO(10) models do. This, besides the required fine-tuning in the doublet sector, is another unsatisfactory aspect of the model.

Extensions of the present model are straightforward. More realistic SO(10) models usually require **120**-plet Higgs to fit the fermion sector [15, 34–36]. By adding a pair of Higgs of **120**-plets with U(1) charges as +1 and −1, respectively, none of the above conclusions fails. The new prediction is $\frac{\alpha_u}{\alpha_d} \tan \beta \sim 10^2$ for the new components from **120**-plet which, again, agrees with the result by simply fitting the data [15, 34–36].

Alternatively, if we use Higgs multiplets in **45**+**54** instead of **210** to break SO(10), proton decay can also be suppressed at the same level. However, since in this case, the **10** Higgs cannot couple with **126** or $\overline{\mathbf{126}}$, to produce the correct contents of the doublets H_u and H_d , a pair of Higgs in **120**-plets are needed to be included at the beginning, because the **120**-plet can both couple with the **10**-plet and the **126**/ $\overline{\mathbf{126}}$ through **45**+**54**.

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